

# POLE AND ZERO ASSIGNMENT TECHNIQUE FOR POWER SYSTEM CONTROLLER DESIGN

A Thesis Submitted  
in Partial Fulfilment of the Requirements  
for the Degree of  
MASTER OF TECHNOLOGY

By  
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to the  
DEPARTMENT OF ELECTRICAL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR  
AUGUST, 1981

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## CERTIFICATE

Certified that this work on 'Pole and Zero Assignment Technique for Power System Controller Design,' by Mr. V.K. Sharma, has been carried out under my supervision and that this has not been submitted elsewhere for a degree.



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August, 1981

## ACKNOWLEDGEMENTS

My sincere thanks are due to my thesis supervisor, Professor S.S. Prabhu whose kind guidance and valuable suggestions at various stages of the thesis work have benefitted me in the successful completion of the work.

I express my thankfulness to my colleague Mr. S.S. Singh for his help in thesis.

At last, I am also thankful to Mr. J.K. Misra for his neat and accurate typing.

VIJAY KUMAR SHARMA

## LIST OF SYMBOLS

System Parameters and Variables:

|          |   |
|----------|---|
| $P$      | generator power output                      |
| $Q$      | generator reactive power output             |
| $B$      | transmission line susceptance               |
| $X$      | transmission line reactance                 |
| $X_d$    | synchronous reactance d-axis                |
| $X'_d$   | transient reactance d-axis                  |
| $\tau_e$ | exciter time constant                       |
| $\tau_s$ | voltage control feedback loop time constant |
| $\tau_g$ | gate time constant                          |
| $\tau_a$ | governor actuator time constant             |
| $\tau_w$ | water time constant                         |
| $\mu_e$  | exciter gain                                |
| $\mu_s$  | voltage control feedback loop gain          |
| $\mu_a$  | governor actuator gain                      |
| $\sigma$ | permanent droop                             |
| $V_o$    | infinite bus voltage, assumed constant      |
| $V_t$    | generator terminal voltage                  |
| $V_f$    | field voltage                               |
| $P_i$    | mechanical power input                      |
| $f$      | field flux linkage                          |
| $\delta$ | torque angle (radians)                      |
| $n$      | speed (rad/sec), after time scaling         |

|            |  |
|------------|--|
| $g$        | gate opening                                   |
| $g_f$      | governor feedback loop signal                  |
| $h$        | water head                                     |
| $\tau$     | scaled time (sec.)                             |
| $o$        | initial value of a variable (subscript)        |
| $'$        | deviation from the initial value of a variable |
| $u_v, u_g$ | control signals (See eqn. 2.1.5)               |
| $G_v, G_g$ | feedback gain rectors                          |

## ABSTRACT

The thesis considers the problem of improving the output response of a power system, consisting of a synchronous generator driven by a hydraulic turbine, having an exciter voltage regulator and a turbine-speed governor, using pole - and zero assignment by state vector feedback.

A numerical problem has been solved and the results have been compared with those obtained earlier by pole-assignment only. The advantages and disadvantages of the present approach have been discussed.

## CHAPTER I

### REVIEW

#### 1.1 Introduction:

One of the most effective methods of controlling a dynamic system is based on the principle of state variable feedback where the input to the system at a given instant, is a linear function of the state of the system at that instant. There are many engineering problems where this type of feedback occurs. For example, the choice of the feedback law may be the direct result of either optimizing a given quadratic performance criterion or reducing sensitivity of the system to plant parameter variations. In still other situations, the feedback law may stem from a desire to achieve arbitrary system dynamics.

It is wellknown [1] that via state-variable feedback it may be possible to arbitrarily alter the dynamics of the system transfer function. Specifically if the system is controllable, state feedback can achieve arbitrary placement of the system poles, subject only to the condition that the number of poles so specified corresponds to the dimension of the controllable space and wherever complex poles occur, they do so in conjugate pairs. The mechanism by which state feedback affects the



zeros of the system is not so clear except in the case of single input and single output systems [2].

The problem of allocating transfer function zeros as well as poles, is currently under investigation, Chen [3], Fallside and Patel [4] and Wang and Desoer [5], have described some procedures. The procedure described by Murdoch [6] enables specified poles and zeros of a scalar transfer function of a controllable and observable linear system to be obtained by using state vector feedback to two inputs. The number of zeros is equal to the number of zeros in transfer function, before feedback is applied, from one input or the other, whichever is greater. Those zeros which can be changed, and other which cannot, are identified. The former can be made equal to, or arbitrarily close to, any assigned values, and the poles can be assigned arbitrarily.

## 1.2 Pole and Zero Assignment Technique:

A linear-time invariant system is described by the equations

$$\dot{x} = Ax + Bu \quad (1.2.1)$$

$$y = c^T x \quad (1.2.2)$$

where  $x$  is an  $n \times 1$  vector,  $u = [u_1:u_2]^T$  is a  $2 \times 1$  input vector and  $y$  is a scalar output,  $B = [b_1:b_2]$ , where the  $n$ -vectors  $b_1$  and  $b_2$  are linearly independent, and the system

is completely controllable through  $u_2$  alone.

The feedback vectors  $k_1^T$  and  $k_2^T$  are found such that the system,

$$\dot{x} = [A + (b_1 : b_2) \begin{pmatrix} k_1^T \\ k_2^T \end{pmatrix}] x + Bu, \quad y = c^T x \quad (1.2.3)$$

has a transfer function between  $y$  and  $u_1$  with as far as possible specified poles and zeros.

The procedure [7] is as follows:

Two sequences of scalars,  $s_1$  and  $s_2$  are formed,

$$s_1 = c^T b_1, \quad c^T A^2 b_1, \quad \dots, \quad c^T A^{p-1} b_1$$

$$s_2 = c^T b_2, \quad c^T A^2 b_2, \quad \dots, \quad c^T A^{q-1} b_2$$

where, in each case, the sequence terminates at the first non-zero term. The method requires that  $q \geq p$ . If this condition is not satisfied, a proportion  $h$  of the input  $u_1$  is added to  $u_2$ , so that  $b_1$  becomes  $(b_1 + hb_2)$ . This will make  $q = p$ . Feedback vectors  $k_1^T$  and  $k_2^T$  are found in two stages.

#### Stage 1:

$k_2^T$  is first determined so as to locate the zeros. Let  $k_1^T$  be a zero vector at this stage. Using a result obtained by Brockett [2], the zeros of the transfer function relating  $y$  to  $u_1$  are eigenvalues of the matrix,

$$\left\{ I - \frac{b_1 c^T (A + b_2 k_2^T)^{p-1}}{c^T (A + b_2 k_2^T)^{p-1} b_1} \right\} (A + b_2 k_2^T) \quad (1.2.4)$$

Since  $q \geq p$ ,

$$c^T (A + b_2 k_2^T)^{p-1} = c^T A^{p-1} \quad (1.2.5)$$

This is proved in Appendix B.

The matrix (1.2.4) may thus be written as,

$$A_o + b_o k_2^T \quad (1.2.6)$$

where,

$$A_o = \left\{ I - \frac{b_1 c^T A^{p-1}}{c^T A^{p-1} b_1} \right\} A$$

$$\text{and } b_o = \left\{ I - \frac{b_1 c^T A^{p-1}}{c^T A^{p-1} b_1} \right\} b_2 \quad (1.2.7).$$

The pair  $(A_o, b_o)$  is checked for controllability, using any method that permits the identification of the uncontrollable eigenvalues. It is proved in Appendix B that  $A_o$  has the eigenvalue 0 of multiplicity at least  $p$ , and that the eigenvalue 0 of multiplicity  $p$  is uncontrollable through  $b_o$ . The remaining  $(n-p)$  eigenvalues of (1.2.6) are the zeros of the transfer function. Of these, any that are uncontrollable through  $b_o$  cannot be changed, whereas all the rest may be assigned arbitrarily by using modal control theory [8,9] to determine  $k_2^T$ . The eigenvalue of multiplicity  $p$  has no physical significance, and

arises only because the degree of the numerator of the transfer function in  $(n-p)$ .

### Stage 2:

The system poles will have been changed by the application of feedback  $k_2^T$ , and  $k_1^T$  is now determined so as to locate the poles as required. It is first necessary to check for controllability of the pair,

$$[(A + b_2 k_2^T), \quad b_1] \quad (1.2.8)$$

If this test is satisfied,  $k_1^T$  may be found by again using modal control theory [8,9] to move the poles to any desired locations. As is well known, the application of the feedback  $k_1^T$  will have no effect on zeros which were established in stage 1, because this feedback is applied to the input from which the transfer function is taken.

If the test for controllability of (1.2.8) fails, controllability can be achieved by making small adjustments to  $k_2^T$ , which means that the zeros in stage 1 can now be made arbitrarily close to assigned values, and not quite equal to them.

### 1.3 Comparison with Other Techniques:

The use of unity rank feedback for the purpose of pole and zero assignment is very restrictive [4, 10], as, with full state vector feedback, a system with  $r$  inputs

can have at most  $(r-1)$  zeros assigned, together with all the poles. This will often be insufficient to permit the desired complete specification of poles and zeros.

Another approach, using state vector feedback of rank 2, [6] permits the assignment of all the poles and zeros of a scalar transfer function, subject to some restrictions, but is limited to the class of systems in which inner product of the input and output vector concerned is non-zero. The method described above is the generalisation of this method.

#### 1.4 On the Zeros of a Linear Multivariable System:

Consider the linear time-invariant system,

$$\dot{x} = Ax + Bu \quad (1.4.1)$$

$$y = Cx + Du \quad (1.4.2)$$

where  $x$  is an  $n$ -vector,  $u$  an  $m$ -vector and  $y$  is a  $p$ -vector. The real matrices  $A, B, C, D$ , are of appropriate dimensions.

The following definitions of system zeros (or transmission zeros) has been proposed in the literature.

(Z 1):

The zeros of  $(A, B, C, D)$  are those complex numbers  $z$  such that,

$$\text{rank} \begin{bmatrix} A-z & B \\ C & D \end{bmatrix} < n + \min(m, p)$$

this definition is due to Davison and Wang [11]. Note that if rank of the polynomial matrix,

$$P(s) = \begin{bmatrix} A-s & B \\ C & D \end{bmatrix}$$

is less than  $n + \min(m, p)$ , then every complex  $z$  is a zero of  $(A, B, C, D)$ .

(Z 2):

Let the rank of  $P(s)$  be  $n+r$ . Let  $\phi(s)$  be the gcd of all  $(n+r)$ -order minors of  $P$  of the form  $[P]_j^i$ , where,

$$i = (i_1, \dots, i_{n+r})$$

$$i_k \in k(k \in n), \quad n < i_{n+1} < \dots < i_{n+r} \leq i_{n+p}$$

$$j = (j_1, \dots, j_{n+r}), \quad 1 \leq j < \dots < j_{n+r} \leq n+m$$

The zeros of  $(A, B, C, D)$  are the complex roots, including multiplicities, of  $\phi(s)$ . This definition is due to Rosenbrock [12].

(Z 3):

The zeros of  $(A, B, C, D)$  are the complex zeros, including multiplicities of the (non-zero) numerator polynomials in the McMillan form of the transfer matrix

$$C(sI - A)^{-1} B + D$$

This definition is also due to Rosenbrock [12], although he calls zeros thus defined the zeros of transfer matrix.

(Z 4):

The transfer matrix can be written,

$$C(sI - A)^{-1} B + D = N(s) D(s)^{-1}$$

where  $N(s)$ ,  $D(s)$  are polynomial matrices unique upto unimodular equivalence. The zeros of  $(A,B,C,D)$  are those complex  $z$  such that,

$$\text{rank } N(z) < \text{rank } N(s)$$

This definition can be attributed to Wolovich [16] and Desoer and Schulman [13].

(Z 5):

This 'geometric' definition applies when  $D = 0$ , suppose that  $A, B, C$  are linear maps, and  $X$  a linear space with  $A : X \rightarrow X$ . Let  $V^*$  be the largest  $(A, B)$ -invariant subspace in  $\text{Ker } C$ , and  $R$  the largest controllability subspace in  $\text{Ker } C$ . Let  $F$  be a linear map such that  $V^*$  is  $(A + BF)$ -invariant (such  $F$  always exists). Finally, let  $\bar{A}_F$  denote the map induced by  $A + BF$  in  $X/R^*$ . The zeros of  $(A,B,C)$  are the roots, counting multiplicities, of the characteristic polynomial of  $\bar{A}_F$  restricted to  $V^*/R^*$ . This definition is due to Morse [14].

It will be convenient to consider two additional definitions.

(Z 6):

The zeros of  $(A, B, C, D)$  are those complex  $z$  such that

$$\text{rank} \begin{bmatrix} A-z & B \\ C & D \end{bmatrix} < \text{rank } P(s)$$

(Z 7):

The zeros of  $(A, B, C, D)$  are the roots, counting multiplicities, of the invariant polynomials of  $P(s)$ .

An essential distinction among the definitions is that (Z 2), (Z 3), (Z 5) and (Z 7) include multiplicities whereas (Z 1), (Z 4) and (Z 6) do not.

The following equivalences have been established.

i) If  $D = 0$  then (Z 5) and (Z 7) are equivalent (Corfmat and Morse [15]).

For the next four equivalences assume  $(A, B)$  controllable and  $(C, A)$  observable.

ii) (Z 3) and (Z 7) are equivalent (Rosenbrock [12]).

iii) If  $D = 0$ , then (Z 3) and (Z 5) are equivalent (Sebakhy [16] and Moore and Silverman [17]).

iv) (Z 4) and (Z 6) are equivalent (Wolovich [13]).

v) Except for multiplicities (Z 3) and (Z 6) are equivalent (Davison and Wang [11]).

In addition, it is easy to see that

vi) Except for multiplicities (Z 6) and (Z 7) are equivalent.



Desoer and Schulman have given a nice physical interpretation of zeros as defined in (Z 4):  $\lambda$  is a zero if some nonzero input proportional to  $\exp(\lambda t)$  does not excite the output. Davison and Wang have indicated the significance of zeros, as defined in (Z 1), for the solvability of a general regulator problem. We adopt the definition (Z 3) because it is the natural multivariable extension of the classical single-input/single-output definition.

### 1.5 Eigenvector Assignment:

In [18] it was stated that '... by a suitable choice of [the state feedback matrix], and hence of some closed-loop eigenvectors, it may be possible to eliminate, ... some of the undesired modes e.g. the slower modes.' This comment was made in notice of the fact that the feedback control law which caused a closed loop multi-input system to have specified eigenvalues is not unique, with different choices yielding identical eigenvalues but different eigenvectors. Since the influence of each eigenvalue on each state variable response depends on the eigenvectors, control of the modal matrix may be as necessary as satisfying eigenvalue criteria [10].

The process of coupling the effect of the individual modes (eigenvalues) to the state variable responses through

the eigenvector entries is nonlinear. This fact becomes apparent if the state variable response is derived by using the modal canonical form. However, it is possible to identify certain desired closed-loop system structures and consequently associated modal structures.

The eigenvectors may, of course, be modified without disturbing pole locations through the non-uniqueness of the modal control process. There is also insufficient freedom to completely select the individual eigenvector shapes. Following is the formulation for eigenvalue/eigenvector assignment for multivariable system, given by Srinath Kumar and Rhoten [19].

Consider the completely - controllable, linear time invariant system,

$$\dot{x} = Ax + Bu \quad (1.5.1)$$

where  $x$  is the state  $n$ -vector,  $u$  is the control  $m$ -vector, and  $A$  and  $B$  are constant matrices of appropriate dimensions. The eigenvalue/eigenvector assignment problem may be stated as follows. Find the state feedback law,

$$u = k^T x \quad (1.5.2)$$

which gives,

$$A + Bk^T = WJV \quad (1.5.3)$$

where  $J$  is the Jordan canonical form with the desired closed loop eigenvalues,  $W$  is the required similarity transformation,

$W^{-1} = V$ , and  $W$  is to satisfy certain constraints. To see what freedom exists in the choice of  $W$ , partition Eqn. (1.5.2) as,

$$\begin{bmatrix} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{bmatrix} + \begin{bmatrix} B_1 \\ \hline B_2 \end{bmatrix} [k_1^T : k_2^T] \\ = \begin{bmatrix} W_{11} & W_{12} \\ \hline W_{21} & W_{22} \end{bmatrix} \begin{bmatrix} J_1 & 0 \\ \hline 0 & J_1 \end{bmatrix} \begin{bmatrix} V_{11} & V_{12} \\ \hline V_{21} & V_{22} \end{bmatrix} \quad (1.5.4)$$

where  $A_{11}, B_1, W_{11}, J_1$  and  $V_{11}$  are  $m \times m$  matrices, the other matrices are compatibly dimensioned, and  $B_1$  is assumed to be nonsingular obtained by, at most, a reordering of the state variables. After some algebraic manipulation Eqn. (1.5.4) reduces to,

$$W_{21} J_1 - P W_{21} = T W_{11} + S W_{11} J_1 \quad (1.5.5)$$

$$W_{22} J_2 - P W_{22} = T W_{12} + S W_{12} J_2 \quad (1.5.6)$$

where  $S = B_2 B_1^{-1}$

$$T = A_{21} - S A_{11}$$

$$P = A_{22} - S A_{12}$$

and the  $V$ 's have been suppressed by use of the expressions for the inverse of a partitioned matrix.

Eqns. (1.5.5) and (1.5.6) represent a set of  $n^2 - nm$  linear equations in  $n^2$  entries of  $W$  and the  $nm$  elements of  $W_{11}$  and  $W_{12}$  can be chosen subject only to the

requirement that  $W$  be nonsingular. This can be accomplished [20] even when  $J_1$  and  $J_2$  have eigenvalues in common with  $P$ . With  $W$  chosen,  $k_1^T$  and  $k_2^T$  are easily evaluated from Eq. (1.5.4).

It is obvious from above analysis that, in general, complete specification of eigenvectors is impossible.

In the present study an attempt was made to develop controller design for a power system by eigenvalue/eigenvector assignment. However, this work could not be completed.

## CHAPTER II

### POWER SYSTEM PROBLEM CHOSEN

#### 2.1 System Model:

The power system, whose controller is to be designed using pole and zero assignment, consists of a synchronous generator driven by a hydraulic turbine, having an exciter voltage regulator and a turbine-speed-governor, as given in Figures (2.1.1) and (2.1.3). The System was considered by Yu et al [21] for which they obtained a feedback control law using the linear optimal regulator theory for quadratic performance index, and Pai et al [22] who obtained a feedback control law using the modal control approach for pole placement.

The linearized equations of the system considered are:

$$\dot{x}(\tau) = Ax(\tau) + Bu(\tau) \quad (2.1.1)$$

where the state vector is,

$$x(\tau) = [\delta \ n \ \psi_f \ v_f \ v_s \ g \ g_f \ h]^T \quad (2.1.2)$$

The A and B matrices are given by,

A =

$$\begin{bmatrix}
 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
 -0.6830 & -0.0346 & -0.0816 & 0.0 & 0.0 & 0.8820 & 1.3200 \\
 -0.0832 & 0.0 & -0.0254 & 0.1370 & 0.0 & 0.0 & 0.0 \\
 -0.1300 & 0.0 & -0.1070 & -0.1370 & -1.3700 & 0.0 & 0.0 \\
 0.0 & 0.0 & 0.0 & 0.0 & -0.2740 & 0.0 & 0.0 \\
 0.0 & -0.0265 & 0.0 & 0.0 & 0.0 & -0.0616 & -1.3700 \\
 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -13.700 \\
 0.0 & 0.0531 & 0.0 & 0.0 & 0.0 & 0.1230 & 2.7400 \\
 & & & & & & -0.1710
 \end{bmatrix}$$

$$B = \begin{bmatrix}
 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
 \end{bmatrix}^T$$

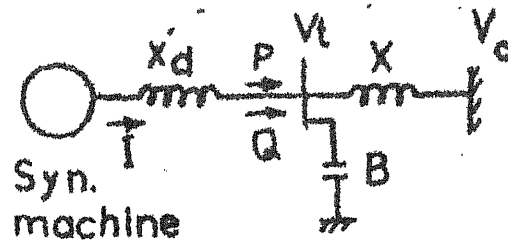


FIG.2.1.1 SYNCHRONOUS MACHINE INFINITE BUS SYSTEM

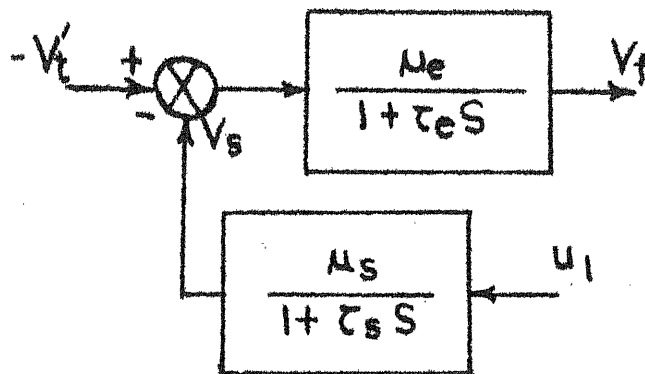


FIG.2.1.2 EXCITER-VOLTAGE REGULATOR SYSTEM

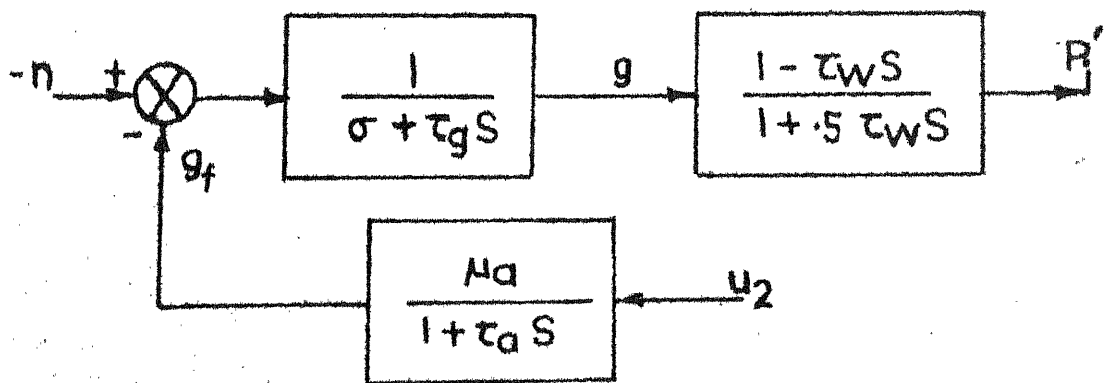


FIG.2.1.3 GOVERNOR-HYDRAULIC SYSTEM

The control vector is given by,

$$u(\tau) = [u_v : u_g]^T \quad (2.1.5)$$

The variable  $\tau$  in Eqn. (2.1.2) is scaled time and is related to the real time  $t$  by the equation

$$\tau = 7.308 t \quad (2.1.6)$$

## 2.2 Numerical Results Obtained by Pai et al:

The modal control configurations considered by M.A. Pai et al, for design are given in Table (2.2.1).

The resulting feedback gain vectors are given in Table (2.2.2).

In the present work the control configuration 1 is chosen for comparing the results obtained by pole and zero assignment technique with those of Pai et al.



Table 2.2.1: Eigenvalues of the power system.

| S.No. | Uncontrolled   | Controlled Configurations                             |                  |                      |
|-------|----------------|---|------------------|----------------------|
|       |                | 1   | 2                | 3                    |
| 1.    | -0.0114        | -0.1952   | -0.1952          | -0.1952              |
| 2.    | $\pm j$ 0.7986 | -0.2740   | -0.2740          | -0.2740              |
| 3.    | -0.0572        | $\begin{Bmatrix} -0.4 \\ \pm j \ 0.915 \end{Bmatrix}$ | -0.6             | $-0.4 \pm j \ 0.915$ |
| 4.    | -0.0772        |   | $\pm j \ 0.8$    |                      |
| 5.    | $\pm j$ 0.1146 | -0.6  | -0.8             | -0.6                 |
| 6.    | -0.1952        | $\begin{Bmatrix} -0.96 \\ \pm j 0.72 \end{Bmatrix}$   | -1.08            | -1.8                 |
| 7.    | -0.274         |   | $\pm j \ 0.5230$ | $\pm j 0.8718$       |
| 8.    | -13.7          | -13.7   | -13.7            | -13.7                |

Table 2.2.2: Feedback gain vectors.

| Modal Control Configurations |         |         |         |          |          |  |
|------------------------------|---------|---------|---------|----------|----------|--|
| 1                            |         | 2       |         | 3        |          |  |
| $G_v$                        | $G_g$   | $G_v$   | $G_g$   | $G_v$    | $G_g$    |  |
| 0.4812                       | 0.8812  | 0.5708  | 1.4505  | 1.3822   | -7.9352  |  |
| 0.1840                       | -5.0268 | 0.2298  | -7.9841 | 0.4687   | -25.4274 |  |
| 1.8181                       | 0.0857  | 1.8014  | 0.1510  | 5.2688   | -1.0508  |  |
| 1.2038                       | -0.1003 | 1.2259  | 0.1551  | 3.2423   | -0.5366  |  |
| -1.7656                      | 0.1831  | -2.0056 | 0.2760  | -3.4456  | 0.4686   |  |
| -6.5651                      | 8.3796  | -6.4963 | 11.0682 | -19.4669 | 50.5043  |  |
| 0.0933                       | -1.3200 | 0.0693  | 1.9200  | 0.2667   | -2.9200  |  |
| -2.7737                      | -2.6776 | -2.8549 | -4.4920 | -8.2778  | 8.9052   |  |

## CHAPTER III

### POSING OF THE PROBLEM

#### 3.1 The Statement of the Problem:

System is described by Eqns. (2.1.1) to (2.1.6). Pole and zero assignment is done for following three cases, with output vectors:

$$\text{i)} \quad c_1^T = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$$

$$\text{ii)} \quad c_2^T = [0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$$

$$\text{iii)} \quad c_3^T = [0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0]$$

The pole locations are chosen to be the same as of conf. 1. in Table (2.1) for all the three cases, so that the results could be compared with those obtained by Pai et al. And zeros are placed such that dominant poles are cancelled, i.e., pole-zero cancellation is tried.

#### 3.2 Numerical Results:

##### Case 1:

The number of zeros is 4 in this case, so they are placed at the locations of the first four dominant poles. While forming series  $s_1$  and  $s_2$  (Sec. 1.2) it was found that  $p > q$ , so  $b_1$  was modified as follows:

$$b_1 = b_1 + 0.5 b_2$$

The final pole-zero locations and the feedback vectors are given in Table (3.2.1).

#### Case 2:

The number of zeros in this case is 5, one of them is uncontrollable. In this case also  $b_1$  is modified as,

$$b_1 = b_1 + 0.5 b_2$$

The results are given in Table (3.2.2).

#### Case 3:

In this case also the number of zeros is 5, all of them are controllable. The results are given in Table (3.2.3).

The transient responses of the controlled system states are shown in Fig. (3.2.1) to (3.2.8).

Table 3.2.1:

| Zeros   |  | Poles  |  | Feedback Gain Vectors                               |  |     |
|---|--|--|--|---|--|-----|
| Uncontrolled Controlled Uncontrolled Controlled |  | Uncontrolled Controlled                                |  | $G_v$   | $G_g$  |     |
| 1.  | -0.1947463   | -0.1951954   | -10.8298900  | -13.7   | 10.450791  | 0.0 |
| 2.  | -0.0441110   | -0.2740036   | -2.9531830   | -0.3999979  | -20.614830   | 0.0 |
| 3.  | $\begin{cases} -0.0623096 \\ -0.3999999 \end{cases}$ | $\begin{cases} -0.01741947 \\ -0.3999999 \end{cases}$  | $\begin{cases} \pm j0.91 \\ \pm j0.91 \end{cases}$     | $\begin{cases} 3.142396 \\ 0.984485 \end{cases}$    | $\begin{cases} -0.368624 \\ -0.417870 \end{cases}$ |     |
| 4.  | $\begin{cases} +j0.4002740 \\ +j0.91 \end{cases}$    | $\begin{cases} +j0.8276846 \\ +j0.91 \end{cases}$      | $\begin{cases} -0.9600032 \\ -0.9600032 \end{cases}$   | $\begin{cases} -0.918442 \\ -72.831365 \end{cases}$ | $\begin{cases} 0.452862 \\ 22.55 \end{cases}$      |     |
| 5.  |  | $\begin{cases} -0.07555680 \\ -0.07555680 \end{cases}$ | $\begin{cases} +j0.7200023 \\ +j0.7200023 \end{cases}$ | $\begin{cases} -4.334313 \\ -0.1604063 \end{cases}$ | $\begin{cases} 0.0 \\ 0.0 \end{cases}$             |     |
| 6.  |  | $\begin{cases} +j0.1011295 \\ +j0.1011295 \end{cases}$ | $\begin{cases} -0.5999981 \\ -0.5999981 \end{cases}$   | $\begin{cases} -36.371058 \\ -0.274 \end{cases}$    | $\begin{cases} 0.0 \\ 0.0 \end{cases}$             |     |
| 7.  |  | -0.1604063   | -0.2739983   |   |  |     |
| 8.  |  | -0.274   | -0.1952008   |   |  |     |

Table 3.2.2

| Zeros                  |                | Poles               |                     | Feedback Gain Vectors |           |
|------------------------|----------------|---------------------|---------------------|-----------------------|-----------|
| Uncontrolled           | Controlled     | Uncontrolled        | Controlled          | $G_v$                 | $G_g$     |
| 1. -0.1947463          | -0.1951954     | -10.82989           | -13.7               | 10.450791             | 0.0       |
| 2. -0.0441110          | -0.2740036     | - 2.953183          | - 0.3999979         | -20.614830            | 0.0       |
| 3. { -0.0623096        | -0.3999999     | { - 0.0174974       | { $\pm j$ 0.91      | 3.142396              | -0.368624 |
| 4. { $\pm j$ 0.4002740 | { $\pm j$ 0.91 | { $\pm j$ 0.8276846 | { - 0.9600032       | 0.98448552            | -0.417870 |
| 5. 0.0                 | 0.0            | { 0.075568          | { $\pm j$ 0.7200023 | -0.91844261           | 0.452862  |
| 6. -                   | -              | { $\pm j$ 0.1011295 | { - 0.5999981       | -72.831365            | 22.55     |
| 7. -                   | -              | - 0.1604063         | - 0.2739983         | -4.3343132            | 0.0       |
| 8. -                   | -              | - 0.274             | - 0.1952008         | 36.371058             | 0.0       |

Table 3.2.3

| Zeros               |                | Poles            |                  | Feedback Gain Vector |                            |
|---------------------|----------------|------------------|------------------|----------------------|----------------------------|
| Uncontrolled        | Controlled     | Uncontrolled     | Controlled       | $G_v$                | $G_g$                      |
| 1. -0.0062861       | -0.1976145     | -13.7            | -13.7            | -0.9351278           | 1.80129                    |
| 2. $\pm j0.8002425$ | -0.2724868     | - 0.400795       | - 0.9600004      | -1.2195051           | -3.84876                   |
| 3. -0.1978477       | -0.3996692     | $\pm j0.9148718$ | $\pm j0.7200001$ | 4.1101374            | 0.0                        |
| 4. -0.0567802       | $\pm j0.90977$ | - 0.263493       | - 0.4            | 1.5990116            | 0.0                        |
| 5. -13.7            | -13.7          | $\pm j0.0497437$ | $\pm j0.9099999$ | -2.08336             | -0.181062x10 <sup>-6</sup> |
| 6. -                | -              | - 0.0516319      | - 0.5999998      | -0.76443076          | 1.56421                    |
| 7. -                | -              | $\pm j0.0975285$ | - 0.1952002      | -0.35639923          | 1.00224                    |
| 8. -                | -              | - 0.274          | - 0.2739997      | -0.84459831          | -4.40952                   |

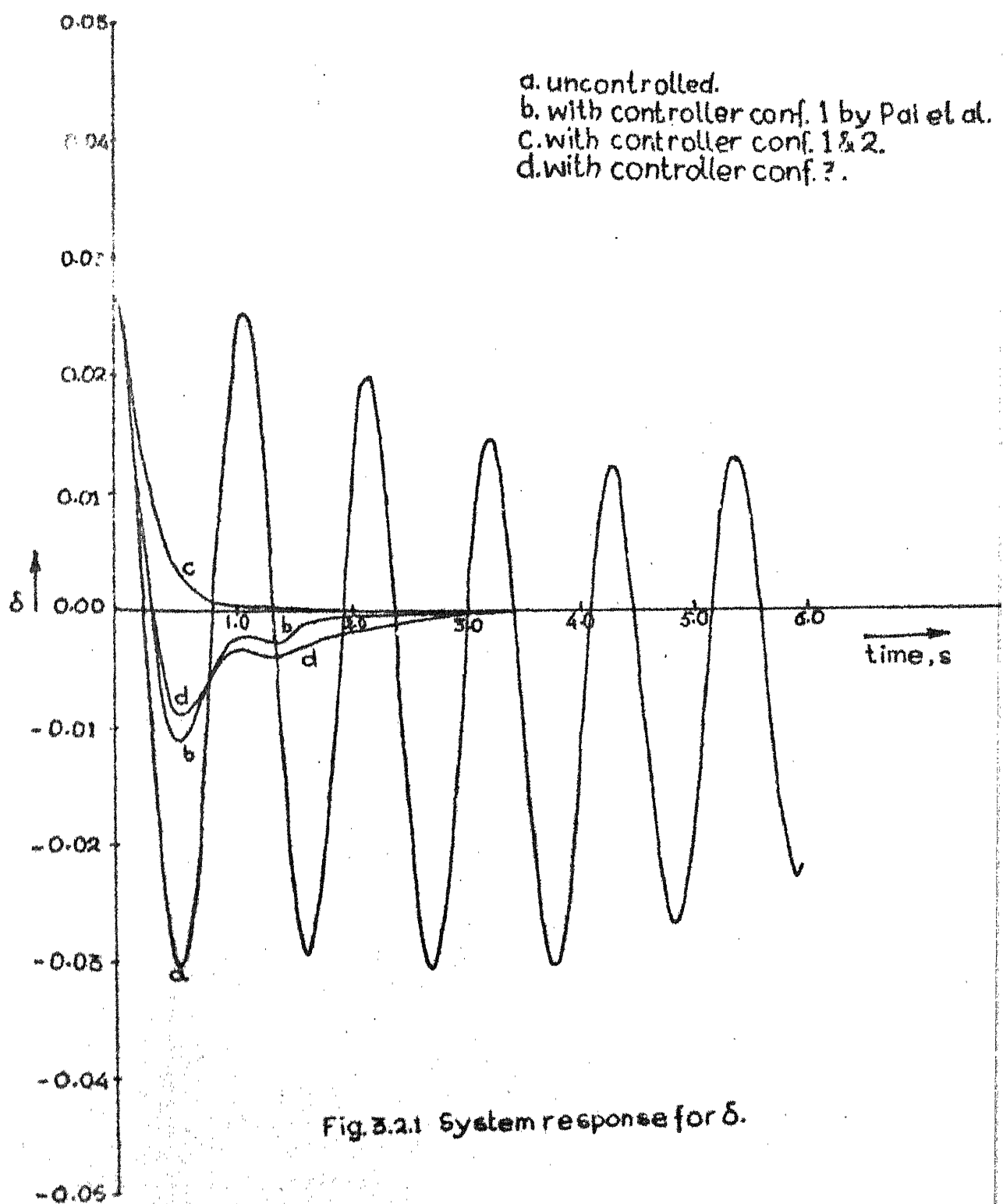


Fig. 3.2.1 System response for  $\delta$ .



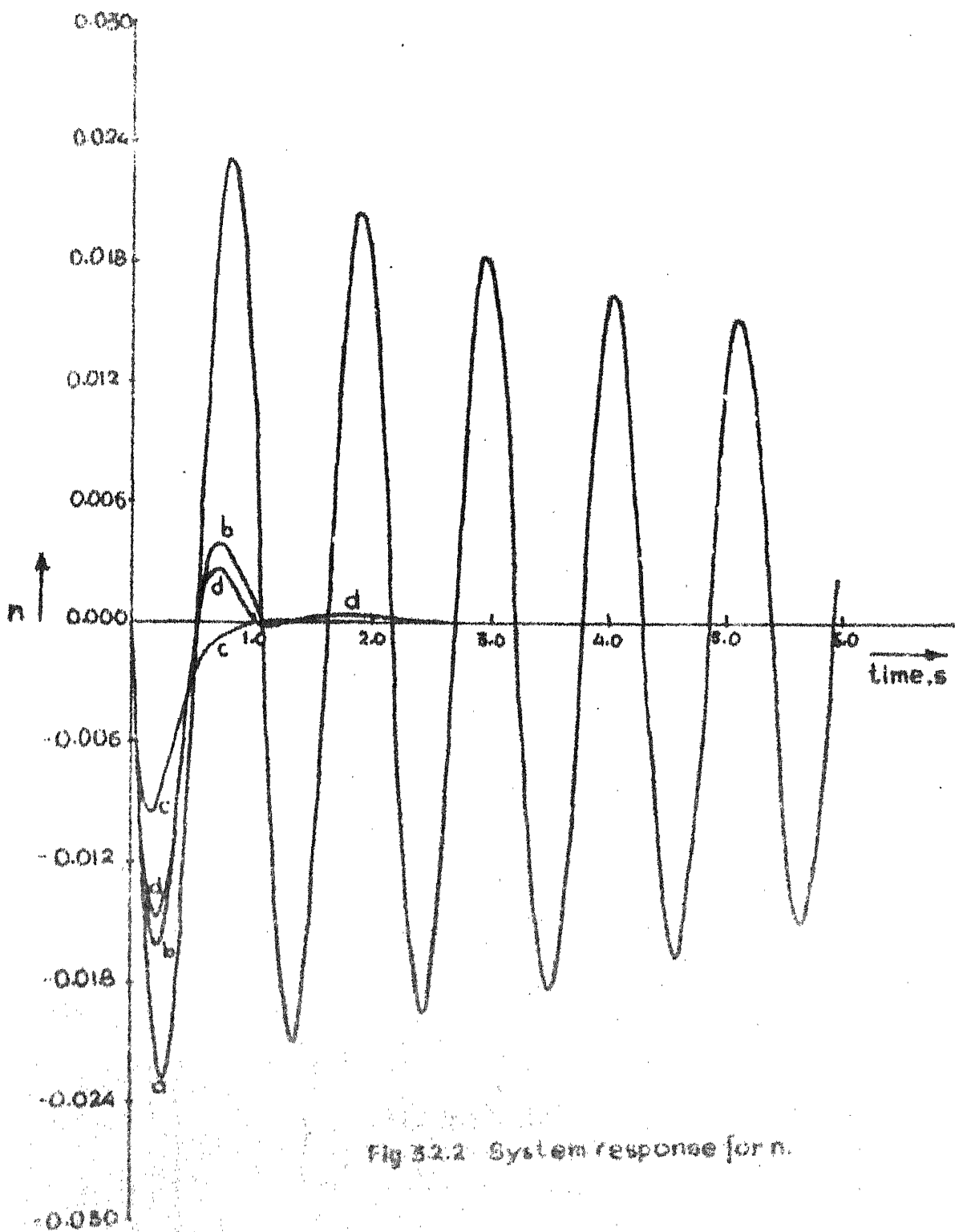


Fig 3.2.2 System response for  $n$ .

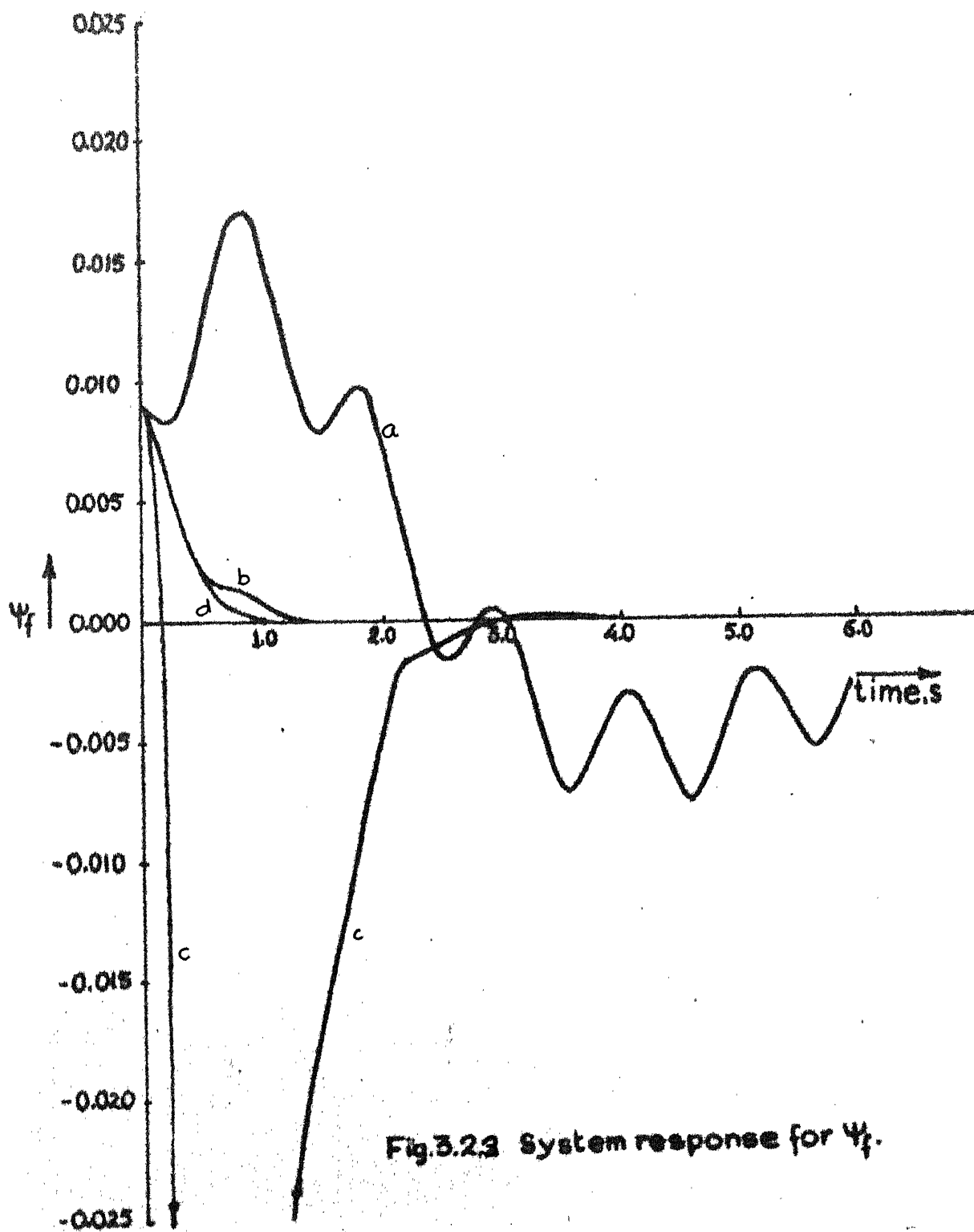


Fig.3.2.3 System response for  $\psi_f$ .

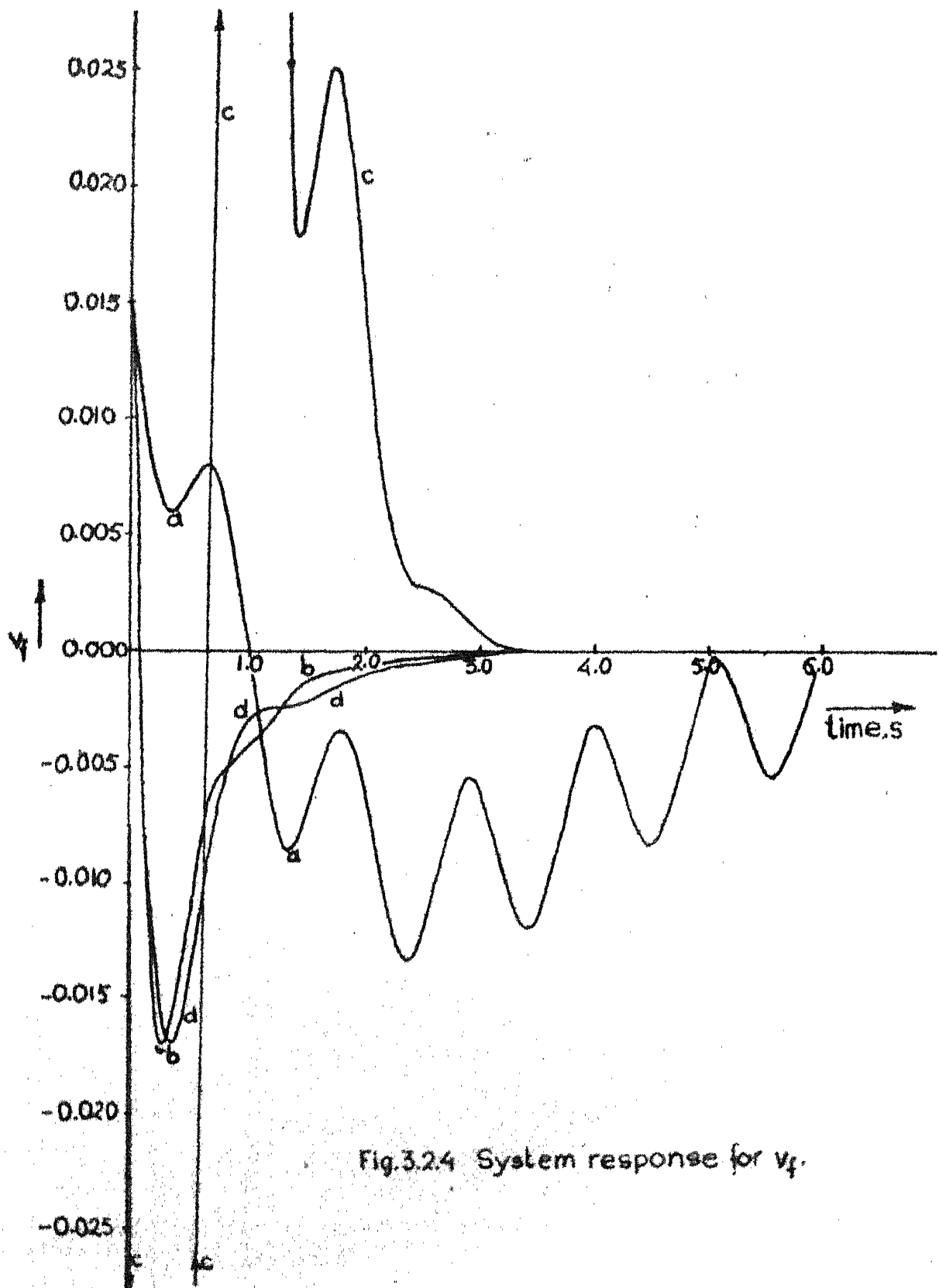


Fig.3.2.4 System response for  $v_f$ .

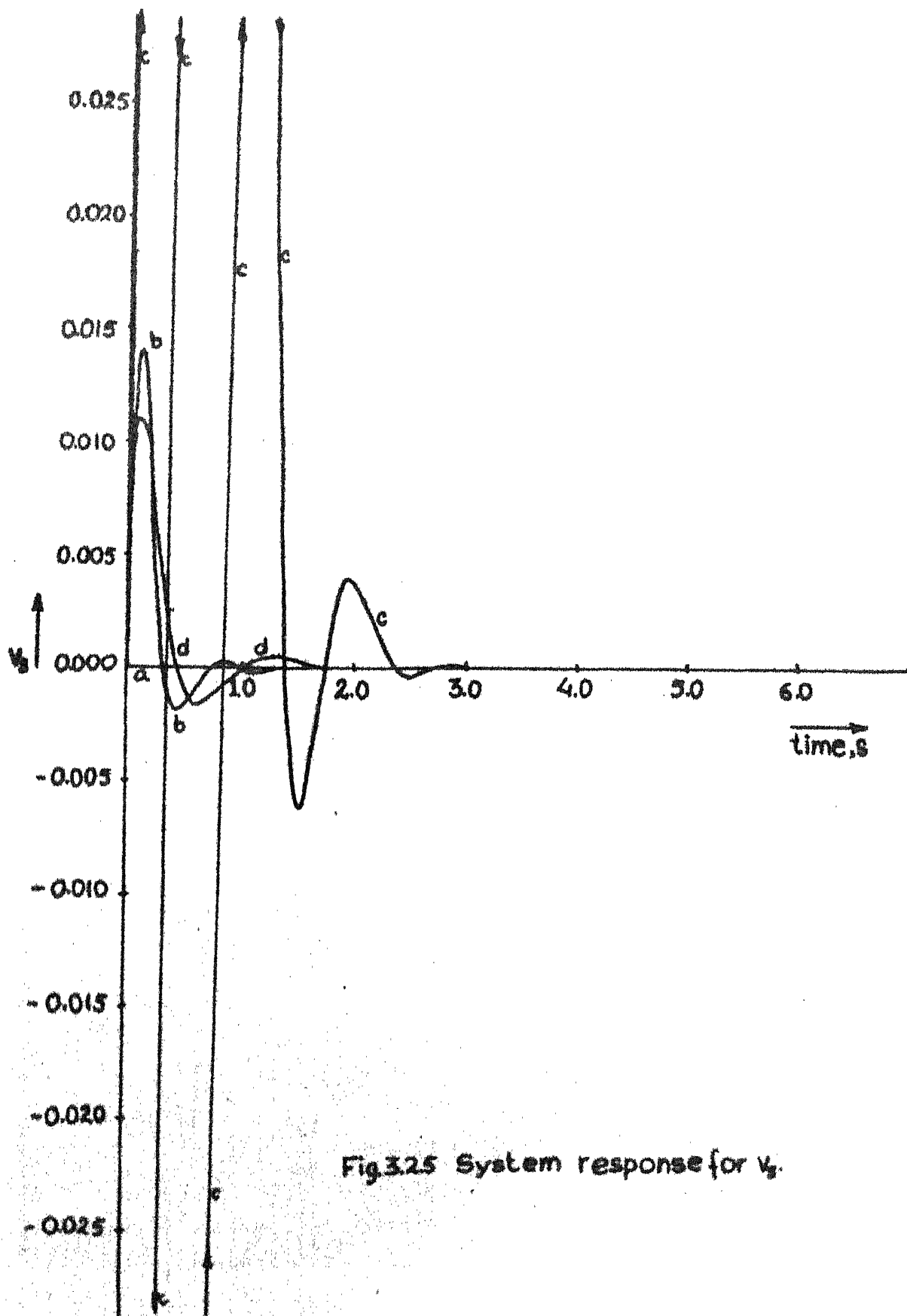


Fig.3.25 System response for  $v_s$ .

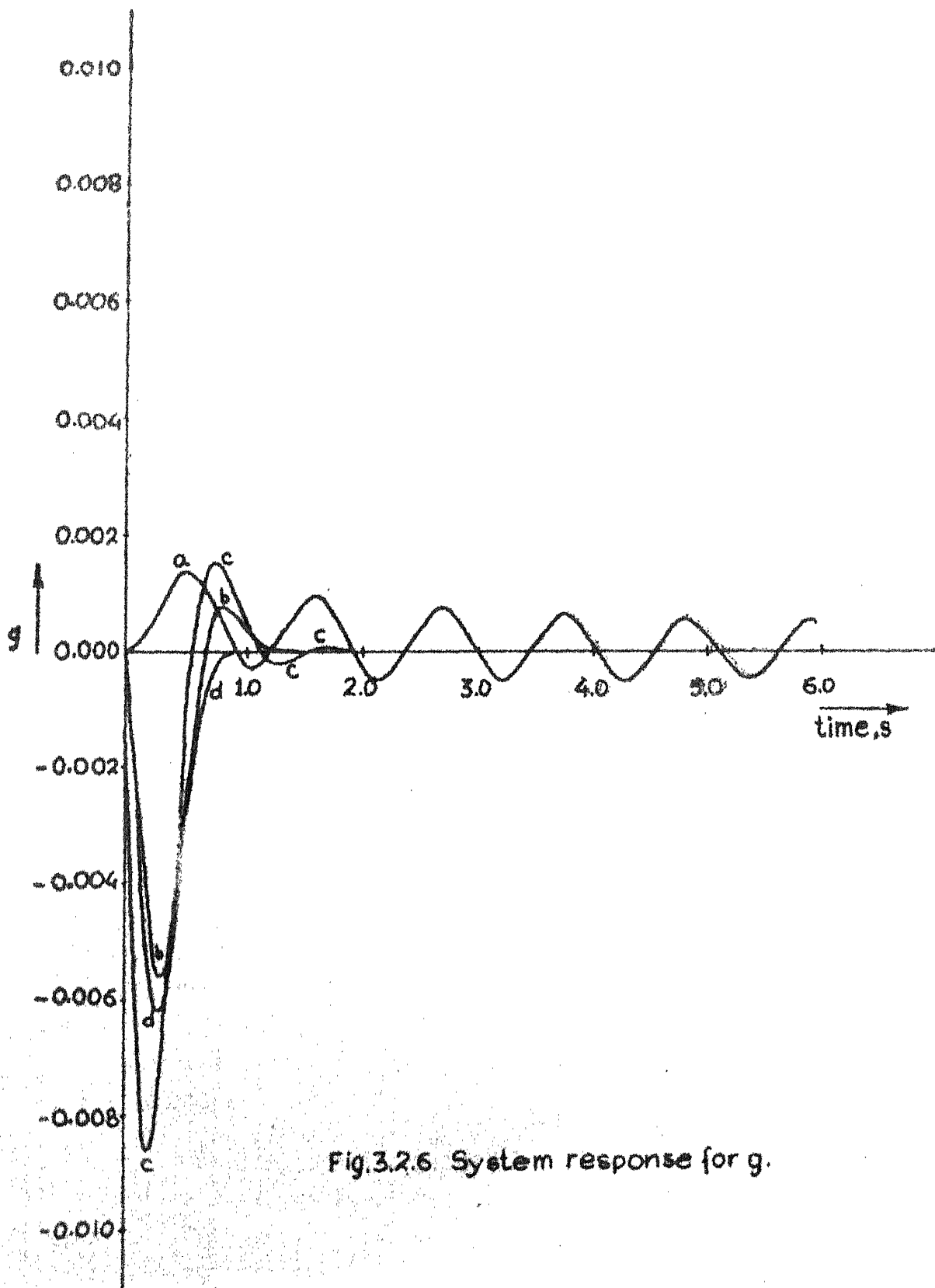


Fig.3.2.6 System response for  $g$ .

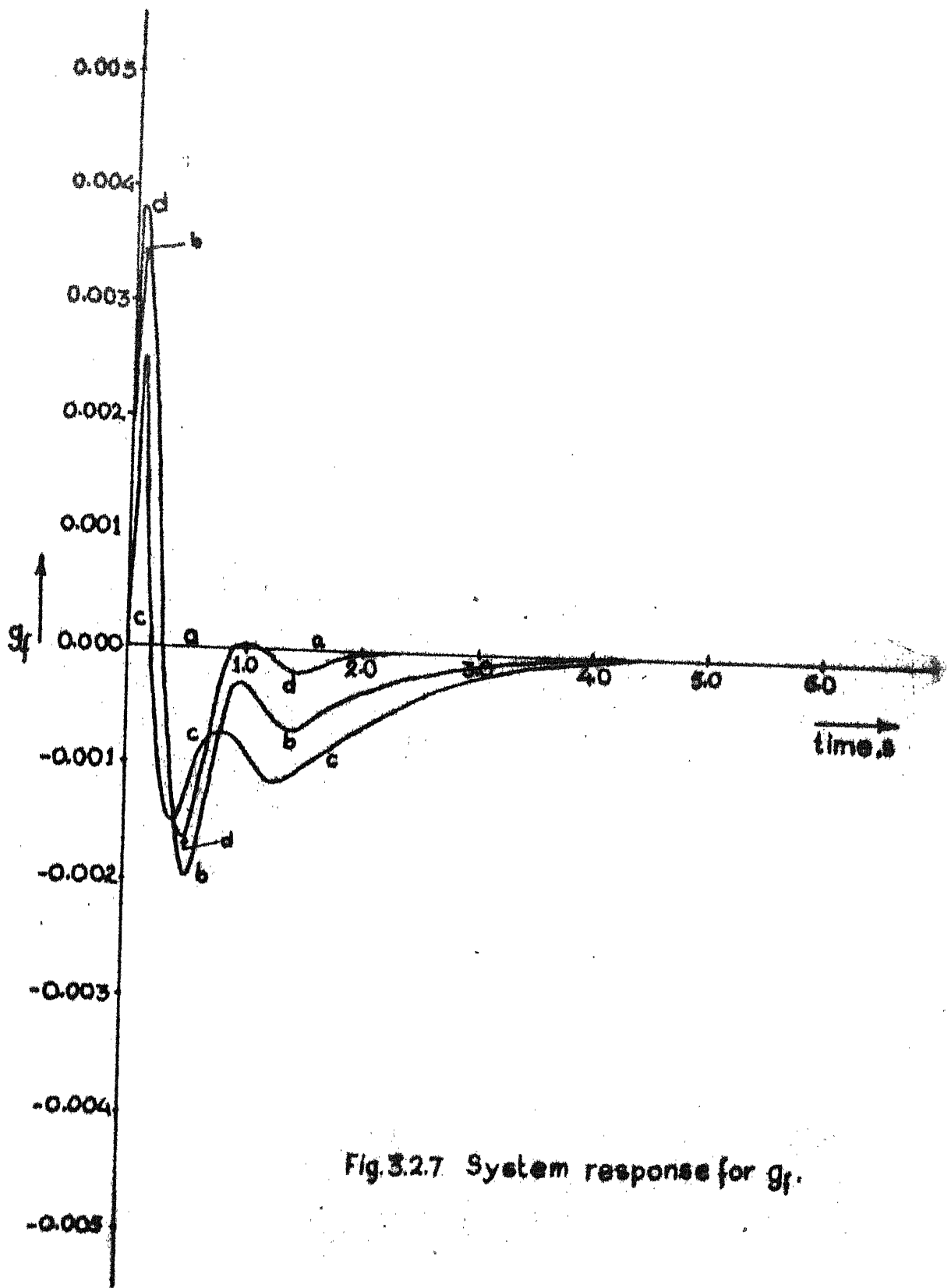


Fig.3.2.7 System response for  $g_f$ .

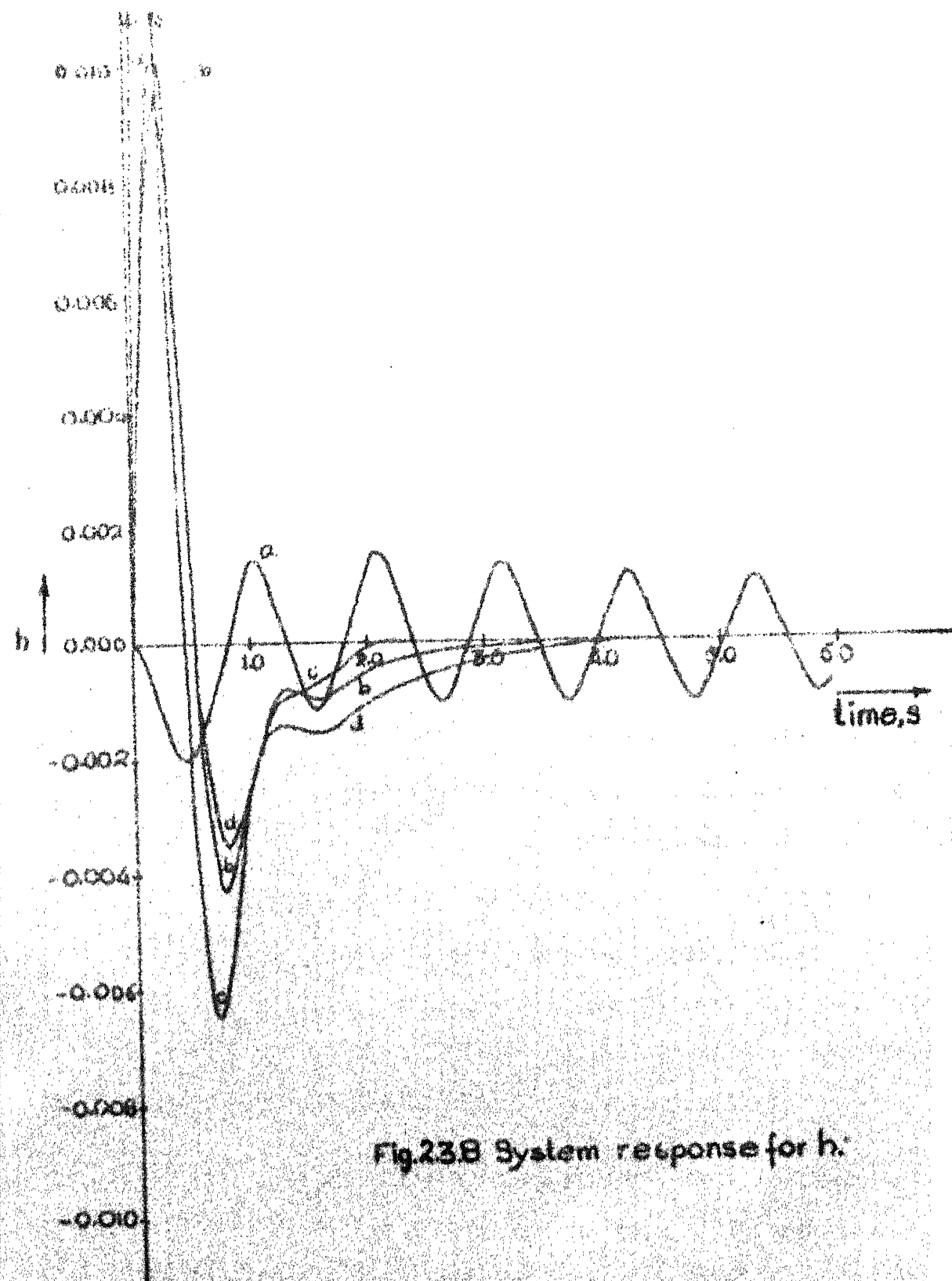


Fig.238 System response for  $h$ .

## CHAPTER IV

### CONCLUSIONS

The feedback controllers for different output vectors are obtained. The poles and zeros of the transfer function relating  $y$  and  $u_1$  are successfully placed. The advantages of this approach over the others are discussed (Sec. 1.3).

Following are the conclusions of study:

- 1) Pole-zero cancellation yielded very good output response, e.g. response corresponding to state variables  $\xi$ ,  $n$  and  $\psi_f$  for output vectors  $c_1^T$ ,  $c_2^T$  and  $c_3^T$  respectively.
- 2) System responses corresponding to different states for control configurations 1 and 2 shows that the response for some states may become bad (e.g. for states  $\psi_f$ ,  $v_f$  and  $v_s$ ) while for other it is quite good.
- 3) There may be some output vector, which will give the satisfactory response for all the states (e.g.  $c_3^T$ ). The responses obtained with  $c_3^T$  are as good as those obtained by Pai et al, and sometimes better.

If the system has more than two inputs,  $u_1$  can be used for zero assignment, other remaining inputs can be used for placing poles. More generally a specified linear combination of a certain inputs can be used for zero placement and rest of the inputs for pole placement.



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## APPENDIX A

### Formation of Matrix (1.2.4):

Consider the linear time-invariant system described by the eqns.

$$\dot{x} = Ax + bu \quad (A1)$$

$$y = c^T x \quad (A2)$$

where  $b$  and  $c$  are  $n$ -vectors.

By repeated differentiation of (A2) and substituting for  $\dot{x}$  from (A1), we obtain a set of equations.  $(y^{(i)} = \frac{d^i y}{dt^i})$

$$\begin{aligned} y^{(0)} &= c^T x \\ y^{(1)} &= c^T Ax + c^T bu^{(0)} \\ y^{(2)} &= c^T A^2 x + c^T Abu^{(0)} + c^T bu^{(1)} \\ &\vdots \\ y^{(n)} &= c^T A^n x + c^T A^{n-1} bu^{(0)} + c^T A^{n-2} bu^{(1)} + \dots + c^T bu^{(n-1)} \end{aligned} \quad (A3)$$

Consider the sequence of scalars  $c^T b, c^T Ab, \dots, c^T A^{n-1} b$ . In general, it cannot be assumed that  $c^T b$  is non zero, or  $c^T Ab$  is nonzero, etc. However, if all the terms in this sequence are zero then clearly  $u$  has no effect on  $y$ , and hence, the transfer function is zero. Assuming that this is not the case, we define the numbers  $\alpha$  and  $q$  as,

$$\begin{aligned} \alpha = i \quad \text{where } c^T A^{i-1} b \neq 0 \text{ and } c^T A^j b = 0 \\ \text{for } 0 \leq j \leq i-1 \end{aligned} \quad (A4)$$

$$\text{and, } q^{-1} = \begin{cases} 0 & \text{for } \alpha = 0 \\ c^T A^{\alpha-1} b & \text{for } \alpha \neq 0 \end{cases} \quad (\text{A5})$$

Both  $\alpha$  and  $q$  have simple interpretation in terms of transfer function. In fact  $\alpha$  is the difference in the degree of numerator and denominator polynomials and

$$q^{-1} = \lim_{s \rightarrow \infty} s^n G(s)$$

We will call  $\alpha$  the relative order of the system.

In terms of this notation, it is seen that  $u$  is given by  $qy^{(\alpha)} - qc^T A^\alpha x$ . Substituting this into (A1), the pair of equations,

$$\dot{x} = (A - bq c^T A^\alpha) x + bq y^{(\alpha)} \quad (\text{A6})$$

$$u = -qc^T A^\alpha x + qy^{(\alpha)} \quad (\text{A7})$$

are obtained. This pair defines what we call as the inverse system.

By taking transform of (A6) and by using the transform of (A7) we arrive at the frequency domain equation,

$$u = -qc^T A^\alpha (sI - A + bq c^T A^\alpha)^{-1} bq s^\alpha y + q s^\alpha y \quad (\text{A8})$$

Since the zeros of the transfer function  $\tilde{y}/\tilde{u}$  are the poles of the transfer function  $\tilde{u}/\tilde{y}$ , and since the latter are the eigenvalues of  $A - bq c^T A^\alpha$ , we see that the zeros of the original system must be eigenvalues of  $A - bq c^T A^\alpha$ .

Now for the closed loop system (1.2.3) we have,

$$u_1 = -\bar{q}c^T \bar{A}^p (sI - \bar{A} + b_1 \bar{q}c^T \bar{A}^{p-1})^{-1} b_1 \bar{q}s^p y + \bar{q}s^p y \quad (A9)$$

where,  $\bar{A} = A + b_2 k_2^T$

and,  $\bar{q} = c^T (A + b_2 k_2^T)^{p-1} b_1$

here it is assumed that  $k_1^T = 0$ . Therefore the zeros of the transfer function  $y/u_1$  (Sec. 1.2) are the eigenvalues of the

$$\begin{aligned} \text{matrix: } \bar{A} - b_1 \bar{q}c^T \bar{A}^{p-1} &= (I - b_1 \bar{q}c^T \bar{A}^{p-1}) \bar{A} \\ &= \left[ I - \frac{b_1 c^T (A + b_2 k_2^T)^{p-1}}{c^T \bar{A}^{p-1} b_1} \right] (A + b_2 k_2^T) \end{aligned}$$

which is the same as (1.2.4).

## APPENDIX B

### a) Proof of Eq. (1.2.5):

By considering vector of the form

$$\begin{aligned} c^T(A + b_2 k_2^T) &= c^T A + c^T b_2 k_2^T = c^T A, \quad \text{if } c^T b_2 = 0 \\ c^T(A + b_2 k_2^T)^2 &= c^T A(A + b_2 k_2^T) = c^T A^2 + c^T A b_2 k_2^T \\ &= c^T A^2, \quad \text{if } c^T b_2 = 0 \text{ and } c^T A b_2 = 0 \end{aligned}$$

and so on, it is easily seen that, since  $c^T A^i b_2 = 0$ ,  
 $i = 0, \dots, q-2$ ,

$$c^T(A + b_2 k_2^T)^i = c^T A^i, \quad i = 0, \dots, (q-1) \quad (B1)$$

eqn. (1.2.5) follows, since  $q \geq p$ .

### b) Proof that $A_0$ has eigenvalue of multiplicity at least $p$ :

The eigenvalues of  $A_0$  are the roots of the characteristic equation,

$$\left| sI - A + \frac{b_1 c^T A^p}{c^T A^{p-1} b_1} \right| = 0 \quad (B2)$$

which may be written,

$$|sI - A| \left| I + \frac{(sI - A)^{-1} b_1 c^T A^p}{c^T A^{p-1} b_1} \right| = 0$$

using the result,  $|I + fg^T| = (1 + g^T f)$ , where  $f$  and  $g^T$  are column and row vectors. respectively. The equation becomes,

$$|sI-A| \left\{ 1 + \frac{c^T A^p (sI - A)^{-1} b_1}{c^T A^{p-1} b_1} \right\} = 0$$

Setting  $s = 0$  makes expression in curly brackets zero. So that 0 is a root. Differentiating the expression in curly brackets with respect to  $s$  and setting  $s = 0$ , makes this expression zero, because  $c^T A^{p-2} b_1 = 0$ . Repeating this operation  $(p-1)$  times, and remembering that  $c^T A^i b_1 = 0$  for  $i = 0, \dots, (p-2)$ , shows that the expression in curly brackets has the root 0 of multiplicity  $p$ , so that  $A_0$  has the eigenvalue 0 of multiplicity at least  $p$ .

c) Proof that in Eq. (1.2.6) the Eigenvalue 0 of Multiplicity  $p$  is Uncontrollable Through  $b_0$ :

The matrix (1.2.4) is of the same form as  $A_0$ , except  $A$  is replaced by  $(A + b_2 k_2^T)$ . It follows from (B1) and from the rules of formation of  $s_1$  and  $s_2$  that the value of  $p$  and  $q$  are unchanged if, in these sequences  $A$  is replaced by  $(A + b_2 k_2^T)$ . Thus by the same argument in (b), the matrix (1.2.4) and hence (1.2.6) has the eigenvalue 0 of multiplicity at least  $p$ , for all  $k_2^T$ . This statement implies that the eigenvalue 0 of multiplicity  $p$  is uncontrollable through  $b_0$ .

## APPENDIX C

Proof that Feedback  $k_1^T$  has no effect on zeros established in Stage 1:

Consider the single input system S,

$$\dot{x} = \tilde{A}x + b_1 u_1 \quad (C1)$$

$$y = c^T x \quad (C2)$$

where  $\tilde{A} = A + b_2 k_2^T$ ,  $x$  and  $y$  are  $n$ -vectors,  $u_1$  is a scalar.

Assume that the system is controllable, in which case without loss of generality, we may represent  $(\tilde{A}, b_1)$  in phase variable canonical form as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

Let the feedback law be given by,

$$u = k_1^T x + v \quad (C3)$$

The system  $\bar{S}$  resulting from feedback has the following form,

$$\dot{\bar{x}} = (A + b k_1^T) \bar{x} + b v$$

$$y = c^T \bar{x}$$

Let  $h(s)$  and  $\bar{h}(s)$  denote transfer functions of  $S$  and  $\bar{S}$  respectively. We have



It is readily verified that,

$$q_i(s) = \bar{q}_i(s) = C_{i1} + C_{i2}s + \dots + C_{in}s^{n-1}$$

for  $i = 1, 2, \dots, n$

Thus numerator polynomial is invariant to state feedback

$k_1^T$ .

$$h(s) = c^T (sI - A)^{-1} b, \quad \bar{h}(s) = c^T (sI - (A + bk_1^T))^{-1} b$$

If  $d(s)$  and  $\bar{d}(s)$  are the characteristic polynomials associated with systems  $S$  and  $\bar{S}$  respectively, then

$$h(s) = \frac{1}{d(s)} [q(s)] \quad \text{and} \quad \bar{h}(s) = \frac{1}{\bar{d}(s)} [\bar{q}(s)]$$

where  $q(s)$  and  $\bar{q}(s)$  are column vectors whose  $n$  entries are polynomials in  $s$ .

Admittedly  $d(s)$  and  $\bar{d}(s)$  are different. We now prove that  $q(s)$  and  $\bar{q}(s)$  are identical irrespective of the choice of the feedback matrix  $k_1^T$ . The proof is as follows:

Since  $b$  is a column vector with all entries zero except the last which is unity, it is easily seen that only the last column of  $(sI - A)^{-1}$  is involved in the product  $(sI - A)^{-1} b$ . Further since  $A$  is a companion matrix, the last column of  $(sI - A)^{-1}$  is given by  $[1, s, s^2, \dots, s^{n-1}]^T$ . Also  $\bar{A} = A + bk_1^T$  is a companion matrix and hence identical arguments show that the last column of  $(sI - \bar{A})^{-1} b$  is given by  $[1, s, s^2, \dots, s^{n-1}]^T$ .

$$\text{Hence } (sI - A)^{-1} b = (sI - \bar{A})^{-1} b = [1, s, s^2, \dots, s^{n-1}]^T$$

Now calculating  $h(s)$  and  $\bar{h}(s)$ , we have

$$h(s) = \frac{1}{d(s)} [q_1(s), q_2(s), \dots, q_n(s)]^T \quad \text{and}$$

$$\bar{h}(s) = \frac{1}{\bar{d}(s)} [\bar{q}_1(s), \bar{q}_2(s), \dots, \bar{q}_n(s)]^T$$